## TABLE 5

### Kinematics of nuclear reactions and scattering

The nonrelativistic expressions for the energetics and solid-angle relations for nuclear reactions and scattering will be found in the accompanying table (overleaf). The material is adapted from the Los Alamos report edited by Jarmie and Seagrave.[1] More complete accounts of nuclear reaction kinematics are available.[2]

Some of the important nonrelativistic and relativistic equations are given below. (The notation is explained in the table overleaf.)

*Nonrelativistic kinematics*

$$\sqrt{E_3} = \frac{\sqrt{M_1 M_3 E_1}}{M_3 + M_4} \cos\psi \left(1 \pm \left\{1 + \frac{1 + M_4/M_3}{\cos^2\psi}\left[\frac{M_4}{M_1}\left(1 + \frac{Q}{E_1}\right) - 1\right]\right\}^{\frac{1}{2}}\right) \tag{5.1}$$

$$E_3 = \frac{M_1 M_3 E_1}{(M_3 + M_4)^2}\left\{2\cos^2\psi + \frac{M_4(M_3+M_4)}{M_1 M_3}\left(\frac{Q}{E_1} - \frac{M_1}{M_4} + 1\right) \pm 2\cos\psi\left[\cos^2\psi + \frac{M_4(M_3+M_4)}{M_1 M_3}\left(\frac{Q}{E_1} - \frac{M_1}{M_4} + 1\right)\right]^{\frac{1}{2}}\right\} \tag{5.2}$$

$$Q = \frac{M_3 + M_4}{M_4} E_3 - \frac{M_4 - M_1}{M_4} E_1 - \frac{2\sqrt{M_1 M_3 E_1 E_3}}{M_4}\cos\psi\,. \tag{5.3}$$

If a particle $x$ with mass $M_1$ produces a $(x, \mathrm{n})$ reaction on a stationary target nucleus of atomic mass $M_2$, the nonrelativistic expression connecting the threshold energy $E_{th}$ and the $Q$-value $Q_0$ is

$$|Q_0| = \frac{M_2}{M_1 + M_2} E_{th} \tag{5.4}$$

where $Q_0 < 0$.

*Relativistic kinematics*

The particles are labeled 1, 2, 3, 4 as in the table on the following page. But now we define

$M$ = rest mass in MeV (i.e., $c = 1$)
$T$ = kinetic energy
$E = T + M$ = total energy
$P$ = relativistic momentum $= \sqrt{E^2 - M^2} = \sqrt{T^2 + 2MT}$
$E_T = T_1 + M_1 + M_2$
$A = 2M_2T_1 + 2M_1M_3 + 2M_2M_3 + 2Q(M_1 + M_2 - M_3) - Q^2$
$B = E_T^2 - P_1^2 \cos^2\psi$
$\dfrac{\partial B}{\partial \psi} = 2P_1^2 \sin\psi \cos\psi$

$$T_3 = E_3 - M_3 = \frac{1}{2B}\left[E_T A \pm P_1 \cos\psi\sqrt{A^2 - 4M_3^2 B}\right] - M_3 \tag{5.5}$$

$$T_4 = E_4 - M_4 = E_T - E_3 - (M_1 + M_2 - M_3 - Q) \tag{5.6}$$

$$Q = M_1 + M_2 - M_3 - \left[M_1^2 + M_2^2 + M_3^2 + 2M_2E_1 - 2E_3(E_1 + M_2) + 2P_1P_3\cos\psi\right]^{\frac{1}{2}} \tag{5.7}$$

$$\zeta = \sin^{-1}\left(\frac{P_3}{P_4}\sin\psi\right) \tag{5.8}$$

$$\frac{\partial T_3}{\partial \psi} = -\frac{E_3}{B}\frac{\partial B}{\partial \psi} \mp \frac{P_1}{2B}\frac{A^2 + 4M_3^2(E_T^2 - 2B)}{\sqrt{A^2 - 4M_3^2 B}}\sin\psi\,. \tag{5.9}$$

The relativistic expression for the threshold energy in terms of $Q$ is

$$E_{th} = |Q|\left(\frac{M_1 + M_2}{M_2} + \frac{|Q|}{2M_2}\right) \tag{5.10}$$

so that in terms of $Q_0$, the nonrelativistic value [see eq. (5.4)], the true $Q$-value is

$$|Q| \cong |Q_0| - \frac{|Q_0|^2}{2(M_1 + M_2)} \tag{5.11}$$

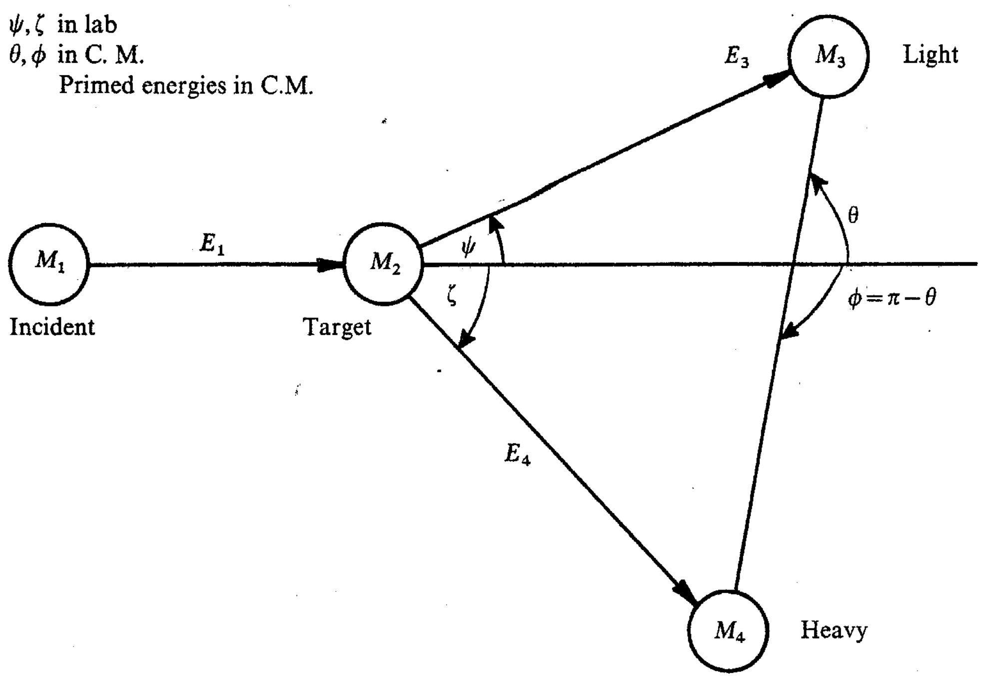
[1] Charged Particle Cross Sections, N. Jarmie and J. D. Seagrave, eds., Los Alamos Scientific Laboratory Report LA-2014, unpublished (1957).

[2] See, for example, A. M. Baldin, V. I. Goldanskiĭ and I. L. Rozental', *Kinematics of Nuclear Reactions* (Oxford University Press, 1961).

TABLE 5

Kinematics of nuclear reactions and scattering (continued)

$\psi, \zeta$ in lab
$\theta, \phi$ in C. M.
Primed energies in C.M.




$Q = (M_1 + M_2 - M_3 - M_4)c^2$
$E_T = E_1 + Q = E_3 + E_4$

Define:

$$A = \frac{M_1 M_4 (E_1/E_T)}{(M_1+M_2)(M_3+M_4)}, \qquad C = \frac{M_2 M_3}{(M_1+M_2)(M_3+M_4)}\left(1 + \frac{M_1 Q}{M_2 E_T}\right) = \frac{E_4'}{E_T}$$

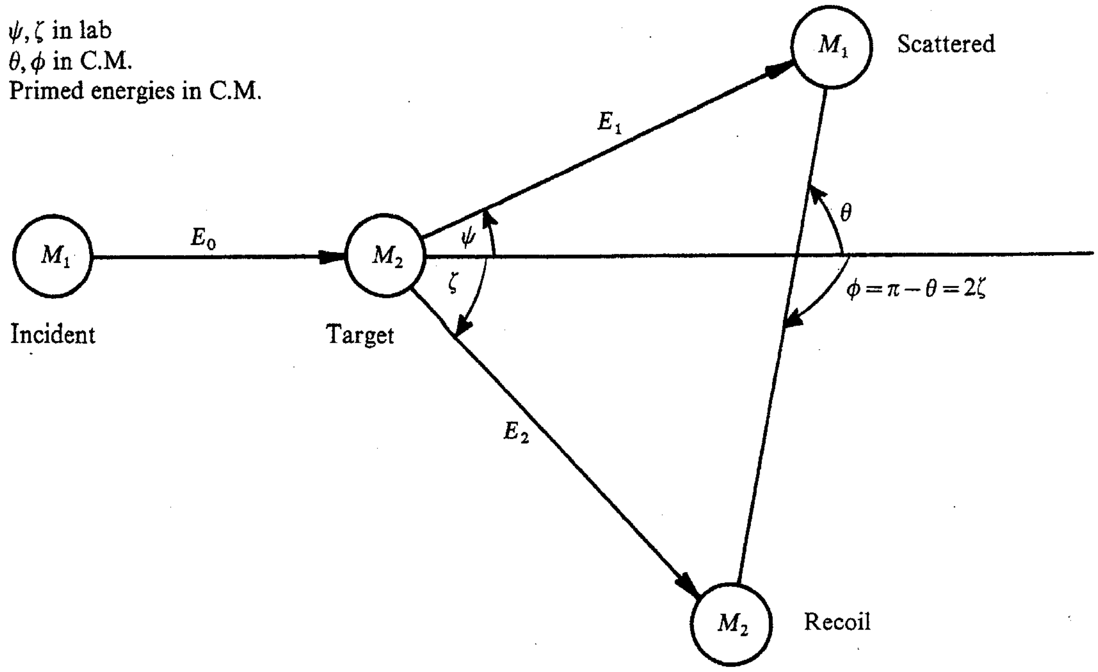
$$B = \frac{M_1 M_3 (E_1/E_T)}{(M_1+M_2)(M_3+M_4)}, \qquad D = \frac{M_2 M_4}{(M_1+M_2)(M_3+M_4)}\left(1 + \frac{M_1 Q}{M_2 E_T}\right) = \frac{E_3'}{E_T}$$

Note that $A+B+C+D=1$ and $AC=BD$

| | | |
|---|---|---|
| Lab energy of light product: | $\frac{E_3}{E_T} = B + D + 2(AC)^{\frac12}\cos\theta$ $= B[\cos\psi \pm (D/B - \sin^2\psi)^{\frac12}]^2$ | Use only plus sign unless $B > D$, in which case $\psi_{max} = \sin^{-1}(D/B)^{\frac12}$ |
| Lab energy of heavy product: | $\frac{E_4}{E_T} = A + C + 2(AC)^{\frac12}\cos\phi$ $= A[\cos\zeta \pm (C/A - \sin^2\zeta)^{\frac12}]^2$ | Use only plus sign unless $A > C$, in which case $\zeta_{max} = \sin^{-1}(C/A)^{\frac12}$ |
| Lab angle of heavy product: | $\sin\zeta = \left(\frac{M_3 E_3}{M_4 E_4}\right)^{\frac12}\sin\psi$ | C.M. angle of light product: $\sin\theta = \left(\frac{E_3/E_T}{D}\right)\sin\psi$ |
| Intensity or solid-angle ratio for light product: | $\frac{\sigma(\theta)}{\sigma(\psi)} = \frac{I(\theta)}{I(\psi)} = \frac{\sin\psi\, d\psi}{\sin\theta\, d\theta} = \frac{\sin^2\psi}{\sin^2\theta}\cos(\theta-\psi) = \frac{(AC)^{\frac12}(D/B - \sin^2\psi)^{\frac12}}{E_3/E_T}$ | |
| Intensity or solid-angle ratio for heavy product: | $\frac{\sigma(\phi)}{\sigma(\zeta)} = \frac{I(\phi)}{I(\zeta)} = \frac{\sin\zeta\, d\zeta}{\sin\phi\, d\phi} = \frac{\sin^2\zeta}{\sin^2\phi}\cos(\phi-\zeta) = \frac{(AC)^{\frac12}(C/A - \sin^2\zeta)^{\frac12}}{E_4/E_T}$ | |
| Intensity or solid-angle ratio for associated particles in the lab system: | $\frac{\sigma(\zeta)}{\sigma(\psi)} = \frac{I(\zeta)}{I(\psi)} = \frac{\sin\psi\, d\psi}{\sin\zeta\, d\zeta} = \frac{\sin^2\psi\cos(\theta-\psi)}{\sin^2\zeta\cos(\phi-\zeta)}$ | |

## TABLE 5

Kinematics of nuclear reactions and scattering (continued)




For elastic scattering, all energy and angle ratios are independent of energy and reduce as below:

$$E_1' = \frac{M_2^2}{(M_1+M_2)^2} E_0$$

$$E_2' = \frac{M_1 M_2}{(M_1+M_2)^2} E_0$$

| | | |
|---|---|---|
| Lab energy of the scattered particle: | $\frac{E_1}{E_0} = 1 - \frac{2M_1M_2}{(M_1+M_2)^2}(1-\cos\theta) = \frac{M_1^2}{(M_1+M_2)^2}\{\cos\psi \pm [(M_2/M_1)^2 - \sin^2\psi]^{\frac{1}{2}}\}^2$ | Use only plus sign unless $M_1 > M_2$, in which case $\psi_{\max} = \sin^{-1}(M_2/M_1)$ |
| Lab energy of the recoil nucleus: | $\frac{E_2}{E_0} = 1 - E_1/E_0 = \frac{4M_1M_2}{(M_1+M_2)^2}\cos^2\zeta \qquad \zeta \leqq \frac{1}{2}\pi$ | |
| Lab angle of recoil nucleus: | $\sin\zeta = \left(\frac{M_1E_1}{M_2E_2}\right)^{\frac{1}{2}} \sin\psi, \qquad \zeta = \frac{1}{2}(\pi - \phi), \qquad \tan\psi = \frac{\sin 2\zeta}{M_1/M_2 - \cos 2\zeta}$ | |
| C.M. angle of scattered particle: | $\theta = \psi + \sin^{-1}\left(\frac{M_1}{M_2}\sin\psi\right) = \pi - 2\zeta \qquad \cos\theta = 1 - 2\cos^2\zeta$ | |
| Intensity or solid-angle ratio for scattered particle: | $\frac{\sigma(\theta)}{\sigma(\psi)} = \frac{I(\theta)}{I(\psi)} = \frac{\sin\psi\,d\psi}{\sin\theta\,d\theta} = \frac{\sin^2\psi}{\sin^2\theta}\cos(\theta-\psi) = \frac{M_1M_2[(M_2/M_1)^2 - \sin^2\psi]^{\frac{1}{2}}}{(M_1+M_2)^2(E_1/E_0)}$ | |
| Intensity or solid-angle ratio for recoil nucleus: | $\frac{\sigma(\phi)}{\sigma(\zeta)} = \frac{I(\phi)}{I(\zeta)} = \frac{\sin\zeta\,d\zeta}{\sin\phi\,d\phi} = \frac{1}{4\cos\zeta}$ | |