lab 7 simple harmonic Motion (SHM)

Pei-Luan Tai

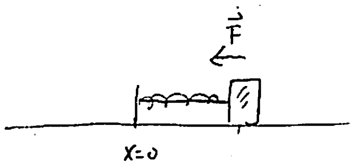
§ restoring force

if our restoring force is $F = -k\underset{\sim}{\Delta x}$

displacement from equilibrium

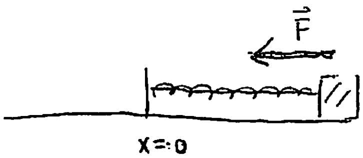
minus sign mean opposite direction

ie



for simplicity we usually write it as

$F = -kx$ or $\vec{F} = -k\vec{x}$



more further from equilibrium point, the restoring force is bigger

$\vec{F} = -k\vec{x}$, for spring, we call it Hooke's law

§ equation of motion

by Newton 2nd law: $\vec{F} = m\vec{a}$, now $\vec{F} = -k\vec{x}$

hence $\vec{F} = m\vec{a} = -k\vec{x}$

for convenience, we deal 1D case, so we drop $\vec{\ }$: vector sign.

instead, we use $+$ to mean $+\hat{x}$, $-$ to mean $-\hat{x}$

$$\underline{\underline{F = ma = -kx}}$$

displacement from equilibrium; more rigor, we should write it as Δx

$$ma = -kx$$

acceleration

------------------------------ derivation with some calculus -----

PS. from calculus:

$$a = \frac{d^2x}{dt^2} \equiv \ddot{x}$$ ... just notation

$$m\ddot{x} + kx = 0$$

$$\ddot{x} + \frac{k}{m}x = 0$$

we define $\frac{k}{m} = \omega^2$

$\ddot{x} + \omega^2 x = 0$, the solution is $x(t) = C_1 \cdot \sin(\omega t) + C_2 \cdot \cos(\omega t)$

need initial condition to determine C_1 C_2 const.

ω is angular frequency; how much radius per time

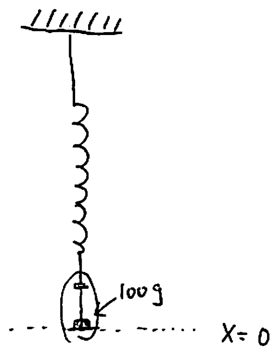
$$\omega = \frac{2\pi}{T} = \frac{1 \text{ circle} = 2\pi \text{ radius}}{1 \text{ circle take } T \text{ sec}}$$

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

ie. time $t=0$, we have no displacement

$$T = 2\pi\sqrt{\frac{m}{k}}$$, and $$x(t) = A\sin(\omega t)$$

Part 1)



in this experiment, we use spring to oscillate vertically.

initially, we hang total 100g at the end of spring

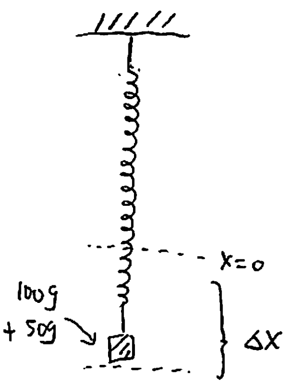
weight hanger 50g

\+ mass 50g

and set the position is our equilibrium point

(reference point)

then add mass, to measure displacement



added mass	ΔX
50 g	5 cm
100g	10 cm
⋮	⋮

Key in your data into template SHM

to check $|F| = K|\Delta X|$; Hooke's law

Part 2

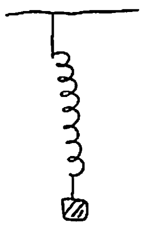
$$T = 2\pi \sqrt{\frac{m}{K}}$$

mass

spring const

period of one oscillation

However, when we let spring oscillate vertically,



we should account the <u>effective mass of spring</u>

m_s = the mass of spring

m_s^* = the effective mass of spring

they have this relation $\underline{m_s^* = \frac{1}{3} m_s}$

the total mass you hang

so.

$$T = 2\pi \sqrt{\frac{m + m_s^*}{K}}$$

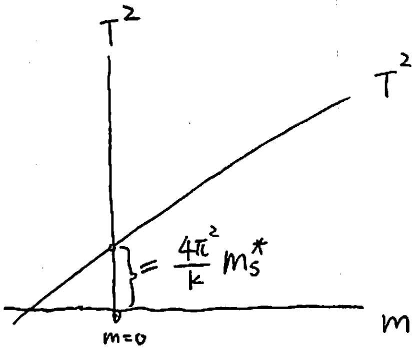
the effective mass of spring

do some manipulation :

$$T^2 = \frac{4\pi^2}{K}(m + m_s^*)$$

$$T^2 = \frac{4\pi^2}{K} m + \frac{4\pi^2}{K} m_s^*$$

in part 2. we plot



$$T^2 = \left(\frac{4\pi^2}{k}\right) m + \frac{4\pi^2}{k} m_s^*$$

from part 1, we know the k

so from the line equation, we can determine m_s^*

our goal is to check

$$\frac{m_s^*}{m_s} \simeq \frac{1}{3}$$

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we oscillate 20 times to record the period

(mass + hanger) total mass	1 try	2 try	avg.
150 g	17.86 sec	18.18 sec	18.02 sec
200 g	19.63	19.96	17.79
250	21.77	21.37	21.57
300	23.76	23.25	23.505
350	25.04	24.70	24.87

from plot, we get

$$T^2 = 3.742\ m + \underline{0.2408} \ ;$$

by $\frac{4\pi^2}{k} m_s^* = 0.2408$, and $k = 9.314$ from part 1

$$m_s^* = \frac{k}{4\pi^2} \cdot 0.2408$$

$$\doteq 0.056\ (kg)$$

$$m_s^* = 56\,g$$

our spring mass m_s is $174.2\,g$

so $\frac{m_s^*}{m_s} \doteq \frac{56}{174.2} \doteq \underline{\underline{0.32}}$ very near to $\frac{1}{3}$

Part 1

Simple Harmonic Motion

Name: Hussein Pei-laun
Lab Partner:
Date: 03/18/2011

Part 1 - Hooke's Law

No	mass m_a (kg)	Force (N)	Displacement (m)	Best Fit Force (N)	Fit Error
1	0.050	0.490	0.050	0.504	-0.0142
2	0.100	0.980	0.100	0.970	0.0101
3	0.150	1.470	0.150	1.436	0.0344
4	0.200	1.960	0.210	1.994	-0.0344
5	0.250	2.450	0.260	2.460	-0.0101
6	0.300	2.940	0.310	2.926	0.0142

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.999645746
R Square	0.999291617
Adjusted R Square	0.999114522
Standard Error	0.027278408
Observations	6

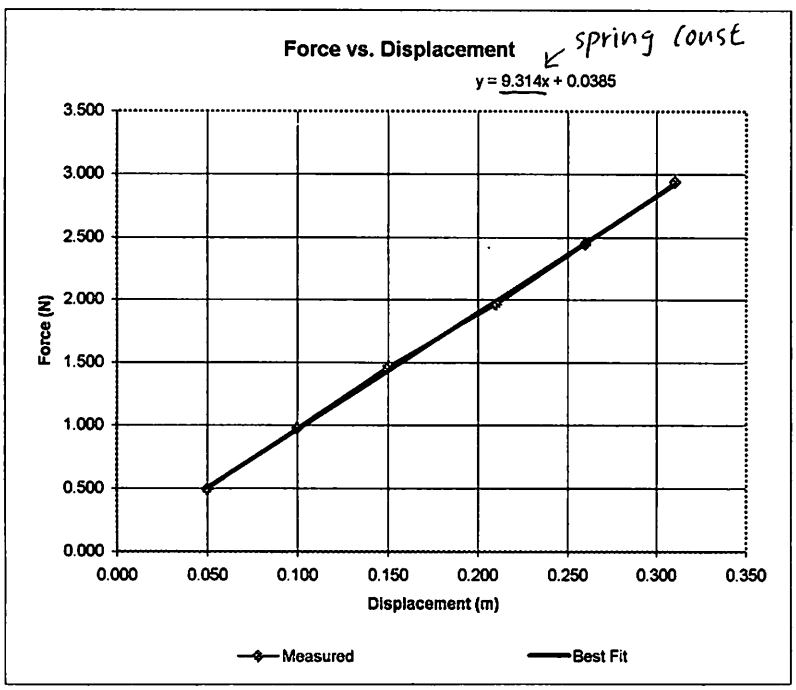
ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	4.198773554	4.198773554	5642.666667	1.88222E-07
Residual	4	0.002976446	0.000744112		
Total	5	4.20175			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.038471074	0.024942792	1.542372421	0.197847522	-0.030781218	0.10772337
X Variable 1	9.314049587	0.123992766	75.11768545	1.88222E-07	8.969790479	9.65830869

RESIDUAL OUTPUT

Observation	*Predicted Y*	*Residuals*
1	0.504173554	-0.014173554
2	0.969876033	0.010123967
3	1.435578512	0.034421488
4	1.994421488	-0.034421488
5	2.460123967	-0.010123967
6	2.925826446	0.014173554



$|F| = k\,|\Delta x|$

Part 2

Simple Harmonic Motion

Name: Hussein , Pei-Luan
Lab Partner:
Date: 03/18/2011

Part 2 - Period^2 versus Mass

No	Mass m_a (kg)	Time for 20 Oscillations (s)	Period T (s)	Period square T^2 (s^2)	Best Fit Period square T^2 (s^2)	Fit Error
1	0.150	18.02	0.901	0.812	0.802	0.0097
2	0.200	19.79	0.990	0.979	0.989	-0.0101
3	0.250	21.57	1.079	1.163	1.176	-0.0132
4	0.300	23.505	1.175	1.381	1.363	0.0178
5	0.350	24.87	1.244	1.546	1.551	-0.0042
6						

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.9989968
R Square	0.9979946
Adjusted R Square	0.9973261
Standard Error	0.0153134
Observations	5

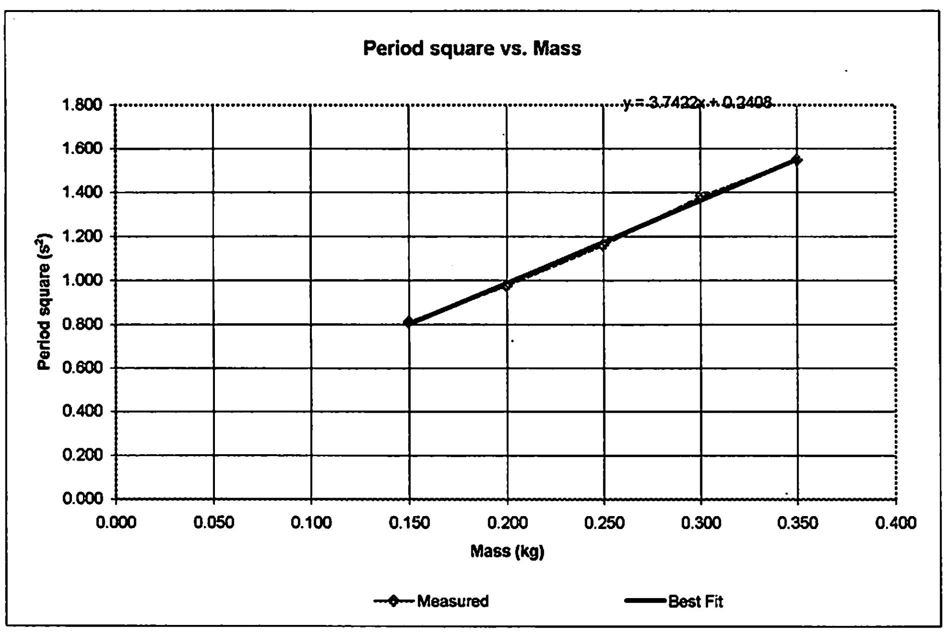
ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.350095838	0.350095838	1492.938515	3.81383E-05
Residual	3	0.000703504	0.000234501		
Total	4	0.350799341			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.2407733	0.025162535	9.568720115	0.002421541	0.160694839	0.3208517
X Variable 1	3.7421696	0.096850643	38.63856254	3.81383E-05	3.433947654	4.0503916

RESIDUAL OUTPUT

Observation	*Predicted Y*	*Residuals*
1	0.8020987	0.0097023
2	0.9892072	-0.010096931
3	1.1763157	-0.013153412
4	1.3634241	0.017788419
5	1.5505326	-0.004240375



$$T^2 = \frac{4\pi^2}{k} m + \frac{4\pi^2}{k} m_s^*$$

Questions:

(1) discuss what happens to the period T, if we consider the air drag. explain your ideas. how much it will change?

(2) the amplitude will affect period T or not ??
(of course you can consider the factor of air drag)
explain your ideas

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(3) if we do the same experiment in the moon. ⇐ (optional problem)

T changes or not ?

explain your ideas.